Chapter 4 Electro-Optic and Magneto-Optic Effects

In this chapter, we are going to discuss the optical response of a dielectric medium under an applied dc electric field, which is the so-called linear and quadratic EO effects.

To facilitate the discussion, let us first consider a non centrosymmetric dielectric crystal, in the presence of an applied dc field $\vec{E}_o$. The optical dielectric function

$\vec{\vec{\varepsilon}}(\omega,\vec{E}_o)$ becomes a function of $\vec{E}_o$,

$$\boxed{\begin{aligned} &\varepsilon(\omega,E_o)=\varepsilon^{(1)}(\omega)+\varepsilon^{(2)}(\omega)E_o+\varepsilon^{(3)}(\omega):E_oE_o+\ldots\ldots \\ &where\quad \varepsilon^{(2)}(\omega)=4\pi\chi^{(2)}\quad \text{(Pockels effect)} \\ &\qquad\qquad \varepsilon^{(3)}(\omega)=4\pi\chi^{(3)}\quad \text{(DC Kerr effect)} \end{aligned}}$$

Dielectric Tensor Approach

The corresponding **index ellipsoid** can be expressed as

$$\boxed{\frac{x^2}{n_{xx}^2(E_o)}+\frac{y^2}{n_{yy}^2(E_o)}+\frac{z^2}{n_{zz}^2(E_o)}+\frac{2yz}{n_{yz}^2(E_o)}+\frac{2zx}{n_{zx}^2(E_o)}+\frac{2xy}{n_{xy}^2(E_o)}=1}$$

Impermeability Tensor approach, where

$$\boxed{\begin{aligned} &\frac{1}{n_{ij}^2(E_o)}=\frac{1}{n_{ij}^2(0)}+\sum_k r_{ijk}E_{ok}\quad \text{with} \\ &\qquad r_{ijk}=\text{linear EO tensor} \end{aligned}}$$

Let us define

$$\eta_{ij}(E_o)\overset{D}{\equiv}\left[\frac{1}{\varepsilon(E_o)}\right]_{ij}\Rightarrow \eta_{ij}(E_o)=\eta_{ij}(0)+\sum_k r_{ijk}E_{0k}+\ldots.,\quad \text{where } r_{ijk}=\left(\frac{\partial\eta_{ij}}{\partial E_{ok}}\right)_{E_o=0}$$

$\boxed{\textbf{Note:}\quad \eta_{ij}=\eta_{ji}\quad\Rightarrow\quad r_{ijk}=r_{jik}}$, implying that $\{ij\}$ can be contracted and be represented by α running from 1 to 6,

$\boxed{r_{ijk}=r_{\alpha k}\qquad \alpha=1,\ 2,\ \ldots,6}$

$\{ij\}$:	xx	yy	zz	yz	zx	xy
α:	1	2	3	4	5	6

4.1 Linear EO Effect

The linear EO effect originates from the charge redistribution of a dielectric medium due to an application of dc electric field.

Note: $r_{\alpha i}$ can be related to the second-order NLO effects.

- For a centrosymmetric crystal, $r_{\alpha i} = 0$
- In the presence of an applied electric field, the index ellipsoid in the principal coordinate system becomes

$$\left(\frac{1}{n_x^2}+\sum_{k=1}^{3} r_{1k}E_k\right)x^2+\left(\frac{1}{n_y^2}+\sum_{k=1}^{3} r_{2k}E_k\right)y^2+\left(\frac{1}{n_z^2}+\sum_{k=1}^{3} r_{3k}E_k\right)z^2$$
$$+2yz\sum_{k=1}^{3} r_{4k}E_k+2zx\sum_{k=1}^{3} r_{5k}E_k+2xy\sum_{k=1}^{3} r_{6k}E_k=1$$

where n_x, n_y, n_z are the principal refractive indices without an external field

Example:

■ KDP ($\bar{4}2m$)

$$\begin{array}{cc} & \begin{array}{ccc} \alpha\downarrow & j\rightarrow & \end{array} \\ r_{\alpha j}(\mathbf{KDP}) = & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52}=r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \end{array}$$

In the presence of an electric field with $\vec{E}=\left(E_x,E_y,E_z\right)$

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_E^2}+2r_{41}E_x yz+2r_{52}E_y zx+2r_{63}E_z xy=1$$

Let $\vec{E}=E(0,0,1)$ to be along $\hat{z}-axis$

$$\frac{x^2+y^2}{n_o^2}+\frac{z^2}{n_E^2}+2r_{63}E_z xy=1.$$ By using a matrix notation, $\vec{r}^{\,T}\overleftrightarrow{A}\,\vec{r}=1$.

$$[x \quad y \quad z]\begin{bmatrix} \frac{1}{n_o^2} & r_{63}E & 0 \\ r_{63}E & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_E^2} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=1$$

Here $\overleftrightarrow{A}$ can be diagonalized by solving its eigenvalue equation,

$$\Rightarrow \left(\frac{1}{n_o^2}-\lambda\right)^2\left(\frac{1}{n_E^2}-\lambda\right)-(r_{63}E)^2\left(\frac{1}{n_E^2}-\lambda\right)=0$$

$$\begin{cases} \lambda_1=\frac{1}{n_o^2}+r_{63}E \\ \lambda_2=\frac{1}{n_o^2}-r_{63}E \\ \lambda_3=\frac{1}{n_E^2} \end{cases}$$ and then deducing the corresponding eigenvectors to assembly an unitary matrix to represent a coordinate transformation

$$\boxed{\overleftrightarrow{U}=\begin{array}{c} \begin{matrix} \hat{e}_1 & \quad\hat{e}_2 & \quad\hat{e}_3 \end{matrix} \\ \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}}$$

Note: by $\vec{r}\,'=\overleftrightarrow{U}\,\vec{r}$

$$\vec{r}^{\,T}\overleftrightarrow{A}\vec{r}=1 \;\Rightarrow\; \vec{r}\,'^{T}\,\overleftrightarrow{U}^T\overleftrightarrow{A}\overleftrightarrow{U}\,\vec{r}\,'=1$$

i.e., $\vec{r}\,'\overleftrightarrow{D}_A\,\vec{r}\,'=1$ where $\overleftrightarrow{D}_A=\overleftrightarrow{U}^T\overleftrightarrow{A}\overleftrightarrow{U}=\textit{the diagonal form of } \overleftrightarrow{A}$.

In the new coordinate system, the index ellipsoid becomes

$$\boxed{\left(\frac{1}{n_o^2}+r_{63}E\right)x'^2+\left(\frac{1}{n_o^2}-r_{63}E\right)y'^2+\frac{z'^2}{n_E^2}=1}\ .$$

The corresponding principal indices of refraction can be found as

$$\boxed{\begin{aligned}
\left(\frac{1}{n_{x'}}\right)^2 &= \frac{1}{n_o^2}+r_{63}E \quad \Rightarrow \quad n_x' \cong n_o-\frac{1}{2}n_o^3 r_{63}E \\
\left(\frac{1}{n_{y'}}\right)^2 &= \frac{1}{n_o^2}-r_{63}E \quad \Rightarrow \quad n_y' \cong n_o+\frac{1}{2}n_o^3 r_{63}E \\
\left(\frac{1}{n_{z'}}\right)^2 &= \frac{1}{n_E^2} \qquad\qquad\;\; \Rightarrow \quad n_z' = n_E
\end{aligned}}$$

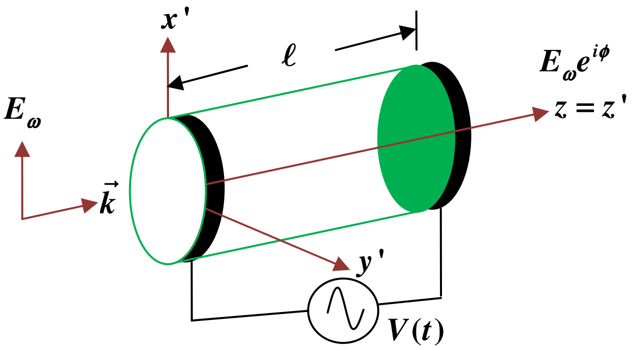
Note $\Delta n = (n_{x'} - n_x) \;\propto\;$ E indicates a linear EO effect

The connection between the **Dielectric Tensor** approach and the **Impermeability Tensor** approach is:

$$r_{ijk} = \frac{-2\chi_{ijk}^{(2)}(-\omega;\omega,0)}{n_i^2 n_j^2}$$

An application example of linear EO effect:

- Phase Modulator



$$n_{x'} = \frac{n_o}{\sqrt{1+r_{63}En_o^2}} \cong n_o - \frac{1}{2}r_{63}En_o^3$$

$$E_\omega(z=\ell) = \frac{V}{\ell} = E_\omega(z=0)e^{i\phi}$$

$$\phi = 2\pi n_{x'}\frac{\ell}{\lambda} = \frac{2\pi \ell n_o}{\lambda}\left[1-\frac{1}{2}r_{63}En_o^2\right] = \phi_0 - \frac{\pi n_o^3 r_{63}}{\lambda}\cdot V(t)$$

4.2 Quadratic Electro-Optic Effect

The quadratic EO effect can occur in a medium with any symmetry.

From $\boxed{\eta_{ij}(E)=\eta_{ij}(0)+S_{ijk\ell}E_kE_\ell \ , \quad \text{where} \ \ S_{ijk\ell}=(\frac{\partial^2\eta_{ij}}{\partial E_k \partial E_\ell})_{E=0}}$

Note:

- $S_{ijk\ell}=S_{jik\ell}$ $\quad (\because \ \eta_{ij}=\eta_{ji})$
- $S_{ijk\ell}=S_{ij\ell k}$ $\quad$ (from the quadratic form of η(E))

$\Rightarrow$ Therefore, we can define a contracted index form of S_{ijkl} as $S_{\alpha\beta}$, where

$$S_{\alpha\beta} \qquad \begin{aligned} \alpha &= 1,2,\ldots\ldots,6 \\ \beta &= 1,2,\ldots\ldots,6 \end{aligned} .$$

The index ellipsoid in the presence of quadratic EO effect becomes

$$x^2\left(\frac{1}{n_x^2}+\sum_{\alpha=1}^{6}S_{1\alpha}E_\alpha^2\right)+y^2\left(\frac{1}{n_y^2}+\sum_{\alpha=1}^{6}S_{2\alpha}E_\alpha^2\right)+z^2\left(\frac{1}{n_z^2}+\sum_{\alpha=1}^{6}S_{3\alpha}E_\alpha^2\right)+yz\sum_{\alpha=1}^{6}S_{4\alpha}E_\alpha^2+\ldots=1$$

- An isotropic medium can be rendered into birefringent in a static electric field. The medium behaves optically as if it is a uniaxial medium in which the electric field defines the optic axis with $\vec{\boldsymbol{E}}=\boldsymbol{E}(\mathbf{0,0,1})$

$$x^2\left(\frac{1}{n^2}+S_{13}E^2\right)+y^2\left(\frac{1}{n^2}+S_{23}E^2\right)+z^2\left(\frac{1}{n^2}+S_{33}E^2\right)=1$$

$$\Rightarrow n_o=n-\frac{1}{2}n^3S_{13}E^2$$

$$n_E=n-\frac{1}{2}n^3S_{33}E^2$$

$$\text{Birefringence}=(n_E-n_o)=\frac{1}{2}n^3(S_{13}-S_{33})E^2=n^3SE^2\propto E^2 .$$

In fact, the quadratic EO Effect belongs to a third-order NLO process. Therefore, it can occur in any medium.

4.3 Physical Properties of EO Coefficients

Assuming $\boldsymbol{q}$, $\boldsymbol{Q}$ be the dynamical variables which describe the electronic (q) and ionic (Q) charge distribution. The impermeability tensor can be expressed as

$\eta = \eta[q(Q)]$.

Adiabatic approximation

(*i.e.*, BO approximation)

***i.e.*, Here we imply electronic potential is mainly determined by the ionic charge distribution. Therefore, a change in the ionic charge distribution will result in a corresponding change in the electronic potential, which in turn changes the polarizability of the solid.**

Consider a modulating field $E(\omega_m)$ being applied on the medium. When the applied frequency nears a characteristic material resonant frequency, a resonance behavior can be observed. Generally speaking, *contributions to the measured EO coefficient will arise from physical mechanisms appropriate to all of the frequencies involved.*

(i) When $\omega_m >> \omega_{ap}$, ω_{op}

optical phonon

acoustic phonon of the medium

This situation occurs when the optical field frequency used is much higher than any lattice resonances and only parametric NLO processes are significant (SHG, OPA, *etc.*)

(ii) When $\omega_m < \omega_{op}$

Optical phonon (relative motion of ion cores inside a unit cell) contributions can not be neglected. In this case,

$$r \equiv \frac{d\chi^{(1)}(\omega_0)}{dE(\omega_m)} = \left[\frac{\partial \chi^{(1)}}{\partial E(\omega_m)}\right]_{Q=0} + \left[\frac{\partial \chi^{(1)}}{\partial Q}\right]_{E=0} \cdot \frac{\partial Q}{\partial E(\omega_m)}$$

$$\textit{where} \quad \left[\frac{\partial \chi^{(1)}}{\partial E(\omega_m)}\right]_{Q=0} \equiv \textit{purely electronic origin from the medium}$$

$$\left[\frac{\partial \chi^{(1)}}{\partial Q}\right]_{E=0} \equiv \textit{the change in the optical susceptibility due to lattice deformation} \sim \boxed{\text{Photoelastic Effect}}$$

$$\left[\frac{\partial Q}{\partial E(\omega_m)}\right] \equiv \textit{action of the modulationg field on the lattice} \sim \boxed{\text{Piezoelectric Effect}}$$

$$\frac{\partial Q}{\partial E(\omega_m)} \sim \frac{(e/M)}{\left(\omega_T^2 - \omega_m^2\right) + 2i\omega_m \Gamma}$$

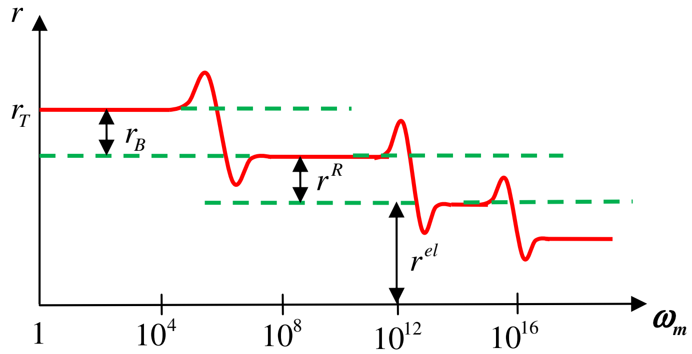
resonant with ω_T (lattice frequency)

The EO effect depends on three distinct processes with the modulating frequency:

(a) $\omega_m < \omega_{ap}$

The **stress** in the medium induced by the applied field can be released via the generation of acoustic phonon. Therefore

$$r_{uk} = r^T = r^\beta + r^R + r^{el} = \text{unclamped} \quad (stress - free)$$



(b) $\omega_{ap} < \omega_m < \omega_{op}$

The **strain** induced by the applied field can be released via the optical phonon generation while the strain can not be relaxed:

$$r_{uk} = r^R + r^{el} = r^S = \text{clamped} \quad (strain - free)$$

(c) $\omega > \omega_{op}$ $\quad r_{uk} = r^{el}$ $\quad$ **Pure electronic orgin** $\quad$ (*parametric* **processes**)

$$\left(\frac{\partial \chi^{(1)}}{\partial E}\right)_{Q=0} = \underline{\left(\frac{\partial \chi^{(1)}}{\partial q}\right)_Q} \left(\frac{\partial q}{\partial E}\right)$$

resonant with $\omega_0 \sim 10^{14} \to 10^{15}\ \text{sec}^{-1}$ (optical frequency)

4.5 Electro-Optic Devices

From the previous discussion, we know that application of the electric field changes the index ellipsoid and therefore the index of refraction of the crystal depends on the field strength of a linearly polarized EM normal mode.

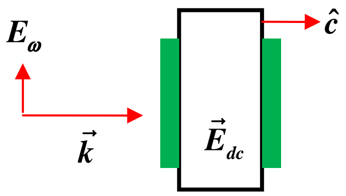
Two Geometries in EO Devices:

- **Transverse:** The applied electric field is perpendicular to the propagation direction

of optical field.
• **Longitudinal:** The d.c. electric field is parallel to the propagation direction of optical field.

■ Longitudinal EO Modulation

(a) z-cut $LiNbO_3$ plate (3m)



$$r(3m)=\begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

$$\vec{E}_{dc} = E\,\hat{z}$$

$$x^2\left(\frac{1}{n_o^2}+r_{13}E\right)+y^2\left(\frac{1}{n_o^2}+r_{13}E\right)+z^2\left(\frac{1}{n_E^2}+r_{33}E\right)=1$$

The principal indexes of refraction with $\vec{E}_{dc} = E\,\hat{z}$ become:

$$\begin{cases} n_x = n_o - \frac{1}{2}n_o^3 r_{13}E \\ n_y = n_o - \frac{1}{2}n_o^3 r_{13}E \\ n_z = n_E - \frac{1}{2}n_E^3 r_{33}E \end{cases}$$

$$\Rightarrow \Delta\phi = \frac{2\pi}{\lambda}n_x L - \frac{2\pi}{\lambda}n_o L = -\frac{\pi}{\lambda}n_o^3 r_{13} V$$

$$V_\pi = \frac{\lambda}{n_o^3 r_{13}} \quad = \text{halfwave voltage } \textit{when } \Delta\phi=\pi$$

No birefringence is induced by the electric field in this device geometry and there is no phase retardation between any two orthogonally polarized waves propagating along $\hat{z}$.

(b) z-cut cubic crystal (GaAs)

Let $\vec{E} = E\,\hat{z}$

$$x^2(\frac{1}{n^2}) + \frac{y^2}{n^2} + \frac{z^2}{n^2} + 2r_{63}Exy = 1 \qquad (r_{63} = r_{41})$$

$$\Rightarrow x'^2(\frac{1}{n^2} + r_{41}E) + y'^2(\frac{1}{n^2} - r_{41}E) + z'^2(\frac{1}{n^2}) = 1$$

$$\left.\begin{cases} n_{x'} = n - \frac{1}{2}n^3 r_{41}E \\ n_{y'} = n + \frac{1}{2}n^3 r_{41}E \\ n_{z'} = n \end{cases}\right\} \Rightarrow \Delta\phi = 2\pi n_{x'}\frac{L}{\lambda} - 2\pi n_{y'}\frac{L}{\lambda} = -2\pi n^3 r_{41}\frac{E}{\lambda}$$

$\Rightarrow$ **An electrical controllable birefringent plate** , which can be used as

• **Phase Modulator (PM)**

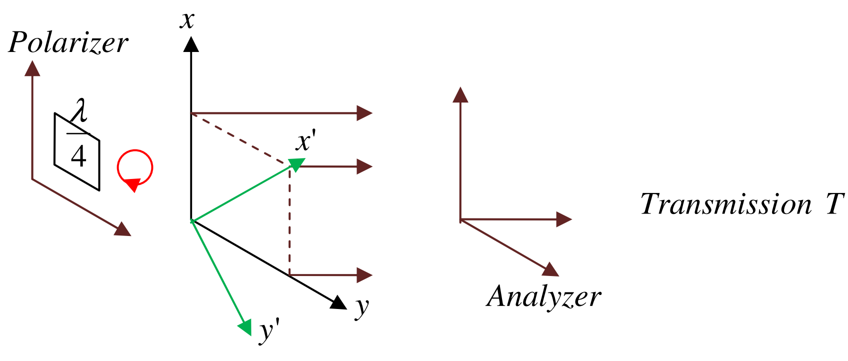
$$\boldsymbol{\Delta\phi = 2\pi n_{x'}\frac{L}{\lambda} - 2\pi n_{y'}\frac{L}{\lambda} = -\pi n^3 r_{41}\frac{V}{\lambda}}$$

$$\boldsymbol{\Rightarrow \quad V_\pi = \frac{\lambda}{n^3 r_{41}} \quad for\ \Delta\phi = \pi}$$ **It is independent of the dimension of the crystal**

To make a pure phase modulator, an input polarizer must be aligned in either the x’ or y’ direction.

• **<u>Amplitude Modulator (AM)</u>**

The input wave can be polarized along the x-axis



$$\Gamma = \frac{2\pi}{\lambda}\left(n_{y'} - n_{x'}\right)L = \frac{2\pi n^3}{\lambda} r_{41} V \propto V_m \sin \omega_m t$$

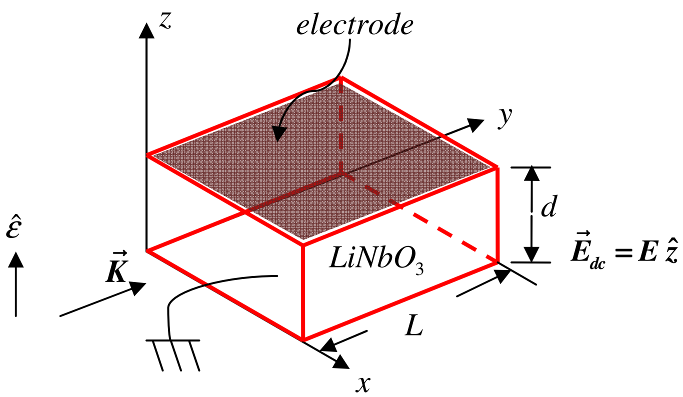
$$V_\pi = \frac{\lambda}{2n^3 r_{41}} \qquad (AM)$$

The optical transmission through the analyzer can be described by:

$$\boldsymbol{T = T_0(1 + \Delta \cdot \sin \omega_m t)} \qquad \boxed{\boldsymbol{where\ \ \Delta = \frac{\pi V_m}{V_\pi}}}.$$

Longitudinal devices have a serious drawback, that is the device designer can not reduce the halfwave voltage by varying the geometrical dimension of EO crystal.

■ **Transverse EO modulation** $(\vec{E}_{dc} \perp \vec{k})$



$$n_z - n_x = (n_E - n_o) - \frac{1}{2}\left(r_{33}n_E^3 - r_{13}n_o^3\right)E$$

$$\Gamma = phase\ retardation = 2\pi\left(n_z - n_x\right)\frac{L}{\lambda} = \frac{2\pi}{\lambda}(n_E - n_o)L - \frac{\pi}{\lambda}\left(r_{33}n_E^3 - r_{13}n_o^3\right)\frac{L}{d}V$$

$$\therefore\quad V_\pi = \frac{d}{L}\cdot\frac{\lambda}{r_{33}n_E^3 - r_{13}n_o^3} \quad \text{depend on } \frac{1}{L} \text{ and } d$$

$$\Delta\phi = phase\ change\ if\ \hat{E} = \hat{z}$$

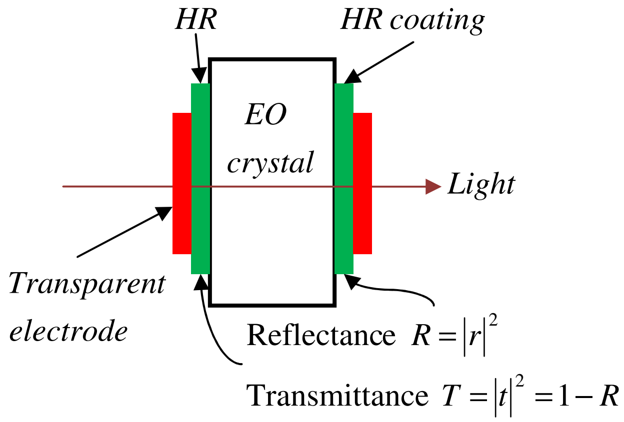
$$= \frac{\pi}{\lambda}n_E^3 r_{33}\frac{L}{d}V = \underline{\pi\left(\frac{V}{V_\pi}\right)} \quad where \quad \boxed{V_\pi = \frac{d}{L}\cdot\frac{\lambda}{n_E^3 r_{33}}}$$

$$\left.\begin{cases} n_{x'} = n - \frac{1}{2}n^3 r_{41}E \\ n_{y'} = n + \frac{1}{2}n^3 r_{41}E \\ n_{z'} = n \end{cases}\right\} \Rightarrow$$

electrical controllable birefringent plate

$$\Delta\phi = 2\pi n_{x'}\frac{L}{\lambda} - 2\pi n_{y'}\frac{L}{\lambda} = -2\pi n^3 r_{41}\frac{E}{\lambda}$$

■ Electro-Optic Fabry-Perot Modulator (Symmetric FP)



From

$$\delta = \textbf{phase modulation depth} = \pi \frac{V_m}{V_\pi}$$

$$\Delta = \textbf{amplitude modulation depth} = \pi \frac{V_m}{V_\pi}$$

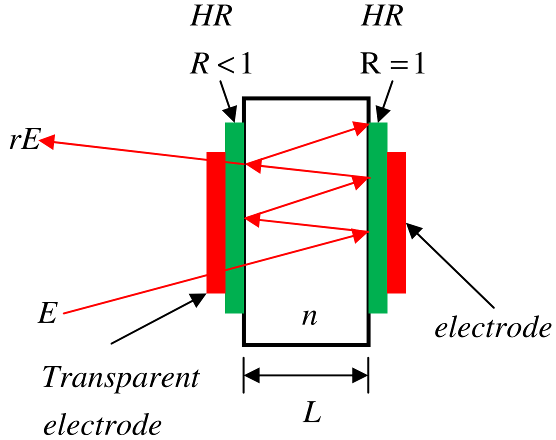
$\Rightarrow$ *if* V_π *small*, Δ *and* δ **are large**

To reduce V_π, long crystal is required. An efficient method to increase crystal is to use a FP cavity to take advantage of the multiple reflected beam.

- **Fabry-Parot Amplitude-Modulation**

Note: T = Transmissivity of the FP cavity $= \boxed{\dfrac{(1-R)^2}{(1-R)^2 + 4R\sin^2\phi}}$

- **FP Phase Modulation (asymmetric FP)**



$$r = e^{i\Phi} = \frac{-\sqrt{R} + e^{-2i\phi}}{1 - \sqrt{R}e^{-2i\phi}} \quad \text{with} \quad r_{12} = -\sqrt{R} \quad r_{23} = 1 \text{ and } \phi = \frac{2\pi nL}{\lambda}$$

$$\Rightarrow \quad \Phi = -2\tan^{-1}\left[\frac{1+\sqrt{R}}{1-\sqrt{R}}\tan\phi\right] \quad \text{where} \quad \phi = \frac{2\pi n_o L}{\lambda} - \frac{\pi}{\lambda} n_o^3 r_{13} V$$

If the device is properly biased such that $\phi(V=0) = m\pi$,

then $\Phi = 2\tan^{-1}\left[\frac{1+\sqrt{R}}{1-\sqrt{R}}\tan(\pi\frac{V}{V_\pi})\right] \cong 2\pi\frac{1+\sqrt{R}}{1-\sqrt{R}}\frac{V}{V_\pi}$ if $V \ll V_\pi$

i.e., Φ can be enhanced by $\frac{1+\sqrt{R}}{1-\sqrt{R}}$ at the expense of the reduction in the bandwidth.

R = mirror reflectivity

ϕ = phase shift = $\frac{2\pi nL}{\lambda} = \frac{2\pi n_o L}{\lambda} - \frac{\pi}{\lambda} n_o^3 r_{13} V$ where $n = n_o - \frac{1}{2} n_o^3 r_{13} E$

then T is electrically tunable.

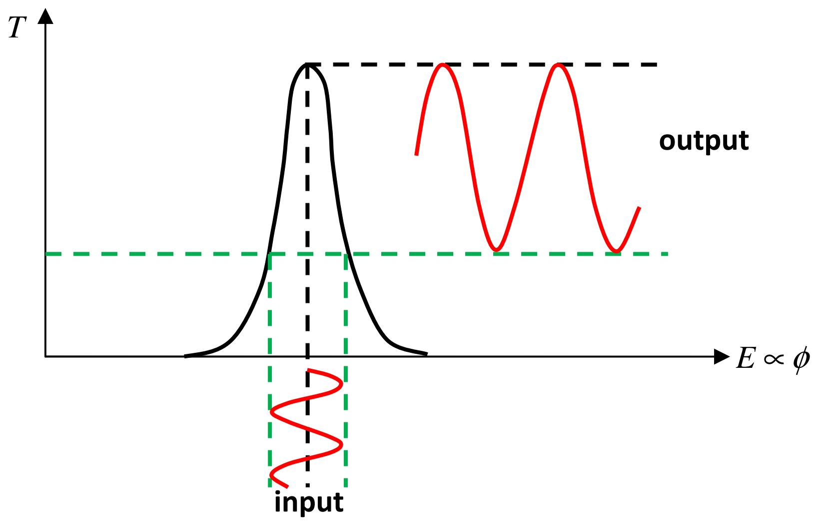
If T is biased at 50% without an applied electric field, then

$$\boxed{\left(\frac{dT}{d\phi}\right)_{1/2} = \frac{F}{\pi} \quad \text{where } F = \frac{4R}{(1-R)^2} = Finesse}$$

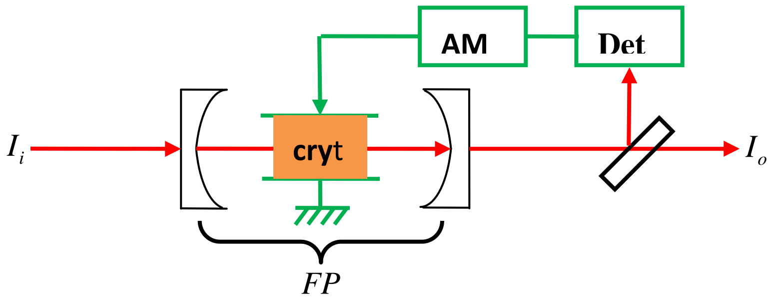
So if F = 30

$$\Rightarrow \left(\frac{dT}{d\phi}\right)_{1/2} \cong 10 \qquad \text{FP}$$

$$\cong 1 \qquad \text{Conventional AM}$$



■ **Bistable EO Devices**

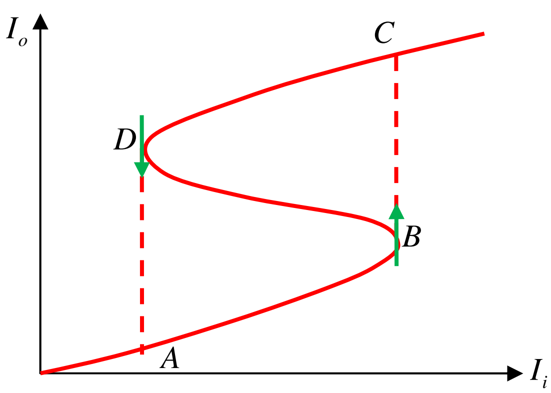


$$T = cavity\ transmissivity$$

$$= \frac{I_o}{I_i} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\phi}$$

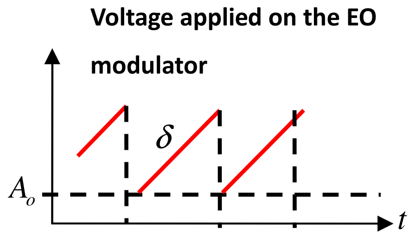
$$where\ \ \phi = \phi_o + \alpha I_o = \frac{1}{2}\ \ (round-trip\ phase\ shift)$$

$$I_i = I_o \frac{(1-R)^2 + 4R\sin^2(\phi_o + \alpha I_o)}{(1-R)^2}$$



■ EO Frequency Shifting & Pulse Compression

Note: if $\Delta\phi \propto V = A_o + \delta t$ *in a EO phase modulator*



Then

$$E(z,t) = E_o e^{i(\omega_o t + \delta t + \phi_o) - ikz} = E_o e^{i\phi(t)}$$

$$i.e., \quad \frac{d\phi(t)}{dt} = \omega = \omega_o + \delta$$

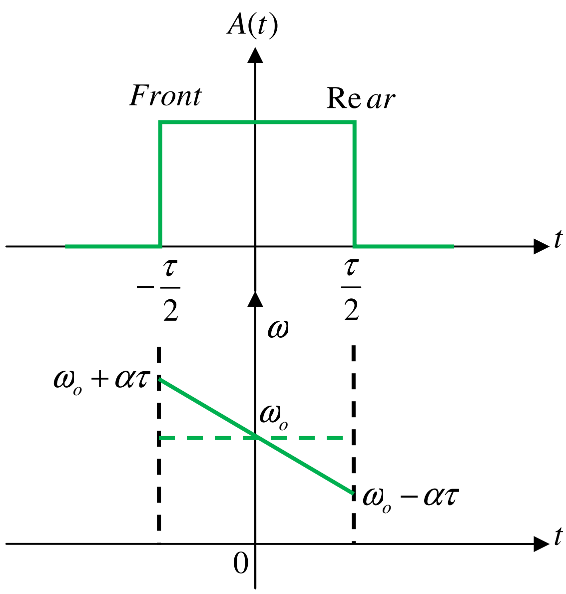
Frequency translation induced by an EO modulator if the driving field is linearly increased its amplitude with a slope of δ.

• if $V = V_0 + \alpha t^2$, the modulating amplitude increases quadratically

$$\Rightarrow E(z,t) = Ae^{i(\omega_o t - \alpha t^2 + \phi_o) - ikz}$$

$$\Rightarrow \omega(t) = \frac{d\phi}{dt} = \omega_o - 2\alpha t$$

Frequency is chirping (i.e., depend on time)

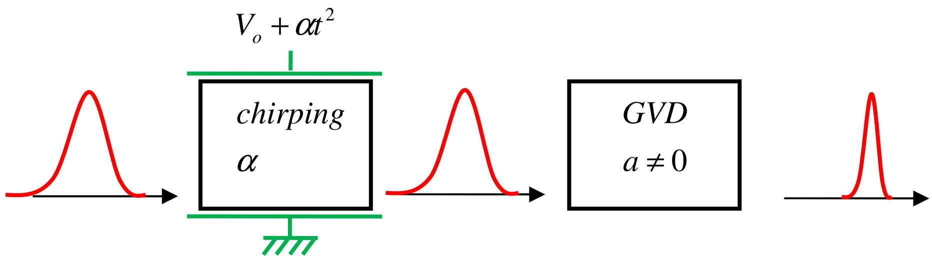


Now let pulses pass through a medium with GVD $\boxed{(a \neq 0)}$

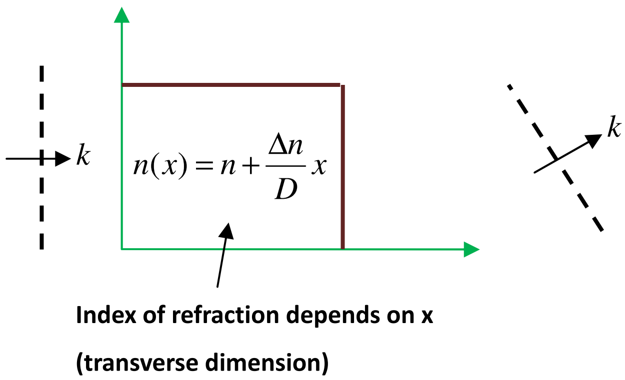
pulse width $\tau \equiv T_f - T_r = \dfrac{L}{v_g(\omega_o + \alpha\tau)} - \dfrac{L}{v_g(\omega_o - \alpha\tau)} = L \cdot \dfrac{d}{d\omega}\left(\dfrac{1}{v_g}\right)\Bigg|_{\omega_o} \cdot 2\alpha\tau$

define $a = \dfrac{1}{2}\dfrac{d}{d\omega}\left(\dfrac{1}{v_g}\right)\Bigg|_{\omega_o}$ = parameter for group velocity dispersion (GVD)

$\Rightarrow if\ \alpha = \dfrac{1}{2L}\dfrac{1}{\dfrac{d}{d\omega}\left(\dfrac{1}{v_g}\right)\Bigg|_{\omega_o}} = \dfrac{1}{4La}$, then unchanged pulse width



■ **EO Beam Deflection**



4.6 Magneto-Optic Effect

Optical dielectric tensor $\boldsymbol{\varepsilon}$ can also be a function of an applied dc magnetic field H_o. However, note that $\boldsymbol{\varepsilon}$ has the following symmetry relation:

I. $\boldsymbol{\varepsilon}_{ij}(\boldsymbol{H}_o) = \boldsymbol{\varepsilon}_{ji}(-\boldsymbol{H}_o)$ since H_o is an axial vector.

II. $\boldsymbol{\varepsilon}_{ij}$ is also Hermitian, *i.e.*, $\varepsilon_{ij}(H_o) = \varepsilon^*_{ji}(H_o)$

Therefore, by using $\varepsilon = \varepsilon' + i\varepsilon''$

II. Hermitian

$\varepsilon_{ij}'(H_o) = \varepsilon_{ji}'(-H_o) = \varepsilon_{ji}'(H_o)$ **:** ε' **is a symmetric and even function of** H_o

I

II. Antisymmetry wrt ij

$\varepsilon_{ij}''(H_o) = -\varepsilon_{ji}''(H_o) = \varepsilon_{ji}''(-H_o)$ **: antisymmetric and odd function of** H_o

I

Here
$\varepsilon_{ij}'(H_o) \Rightarrow$ linear birefringence (Cotton - Mouton Effect)
$\varepsilon_{ij}''(H_o) \Rightarrow$ circular birefringence (Faraday Effect)

Example:

Consider a medium of uniaxial symmetry, having $\vec{H}_o$ parallel to the z-axis:

Real part: $\varepsilon_{xx}' = \varepsilon_{yy}'$, ε_{zz}' is even in H_o

$$\boxed{\vec{\vec{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx}' & i\varepsilon_{xy}'' & 0 \\ -i\varepsilon_{xy}'' & \varepsilon_{yy}' & 0 \\ 0 & 0 & \varepsilon_{zz}' \end{pmatrix}}$$

Imaginary part: $\varepsilon_{xy}'' = -\varepsilon_{yx}''$ odd in H_o

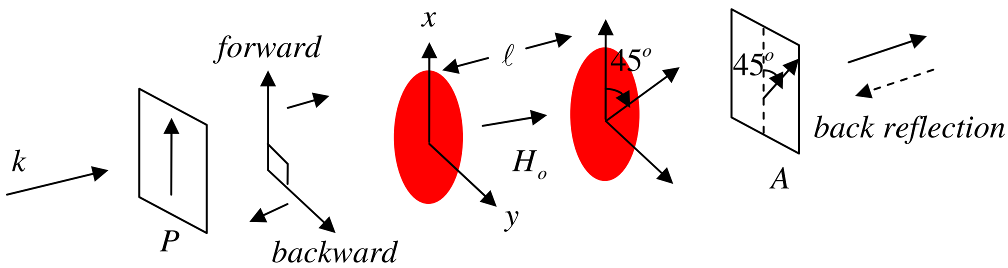
Diagonalize ε with $\hat{e}_\pm = \dfrac{(\hat{x} \pm i\hat{y})}{\sqrt{2}}$ and $\hat{z}$ as the basis yields:

$$\boxed{\varepsilon_\pm = (\varepsilon_{xx}' \pm \varepsilon_{xy}'') \text{ , } \varepsilon_{zz}}$$

Since $|\varepsilon_{xy}''| << |\varepsilon_{xx}'| \Rightarrow k_\pm = \dfrac{\omega\sqrt{\varepsilon_\pm}}{c} = \dfrac{\omega\sqrt{\varepsilon_{xx}'}}{c\varepsilon_{xx}'}\left(1 \pm \dfrac{1}{2}\varepsilon_{xy}''\right)$

The circular birefringence in a medium of length l: $\Delta k\, l = (k_+ - k_-)l = \dfrac{\omega\, \varepsilon_{xy}''}{c\varepsilon_{xx}'} l$.

A linearly polarized beam propagating along $\hat{z}$ will have its polarization rotated by an angle: $\phi = (k_+ - k_-)l$



Forward Propagation:

Input on the medium

$\hat{e}_x = \frac{1}{\sqrt{2}}(\hat{e}_+ + \hat{e}_-)$=linearly polarized beam can be decomposed into two fields with + and – circular polarizations which are the eigen-modes of the MO medium.

After propagates a distance ℓ

$$\begin{aligned}\hat{e}_x \xrightarrow{\ell} &\frac{1}{\sqrt{2}}\left(\hat{e}_+ e^{ik_+\ell} + \hat{e}_- e^{ik_-\ell}\right) \\ &= \frac{1}{\sqrt{2}} e^{ik_+\ell}\left(\hat{e}_+ + \hat{e}_- e^{-i\phi}\right) \\ &= \frac{1}{\sqrt{2}} e^{ik_+\ell}\left(\frac{\hat{e}_x + i\hat{e}_y}{\sqrt{2}} + \frac{\hat{e}_x - i\hat{e}_y}{\sqrt{2}} e^{-i\phi}\right)\end{aligned}$$

$$if \quad \phi = \frac{\pi}{4} \Rightarrow \frac{1}{2} e^{ik_+\ell} \cdot \left[\hat{e}_x + \hat{e}_y\right] \cdot 2(1+i)$$

i.e., A linearly polarized beam with 45^o rotation